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Quantum communication between trapped ions through a dissipative environment

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Abstract
We study two trapped ions coupled to the axial phonon modes of a one-dimensional Coulomb crystal. This system is formally equivalent to the 'two spin-boson' model with position-dependent couplings. We propose a scheme to dynamically generate a maximally entangled state of two ions within a decoherence-free subspace. Here the phononic environment of the trapped ions, whatever its temperature and number of modes, serves as the entangling bus. The production of the pure singlet state can be exploited to perform short-ranged quantum communication which is essential in building up a large-scale quantum computer.

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(Some figures in this article are in colour only in the electronic version)

The spin-boson model plays an important role in studying open quantum systems in physics and chemistry [1, 2]. Recently, Porras *et al* [3] have proposed to use a trapped ion coupled to a set of collective modes of a one-dimensional (1D) Coulomb crystal such that the spin-boson model can be realized and studied precisely. This model can be physically realized by shining a laser on an ion in which two internal states are coupled to a traveling wave [3]. The axial motional modes of 1D Coulomb crystal act as a phonon bath and provide the Ohmic spectral density. This paves the way for the experimental studies of the Ohmic spin-boson model in the low- as well as high-temperature regimes.

In fact, sophisticated techniques have been demonstrated in the manipulation of trapped ions such as cooling the ions to the motional ground state and detecting the state of the ions [4]. Besides, Coulomb crystals of ion gases have been observed in Paul [5] and Penning traps [6] and storage rings [7]. A few dozens of ions separated by several micrometers in a crystal form have been observed. This provides a promising ground to investigate the spin-boson model with trapped ions.

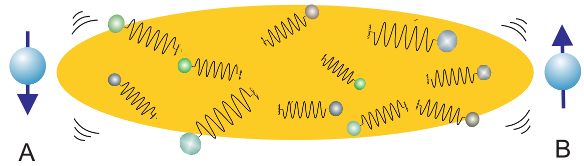

Figure 1. Two ions are coupled to a set of axial phonon modes of the Coulomb chain which is acted as a harmonic bath.

The work of this paper will be based on the 'two spin-boson' model which can be implemented in a 1D Coulomb crystal. The two spin-boson model is a generalization of the scheme of the ion-trap spin-boson model of Porras *et al* [3]. The two ions are now considered to couple to a set of axial phonon modes of the chain. This phonon bath plays the role of a common dissipative environment for the two ions (see figure 1). Nevertheless, the decoherence can be greatly canceled provided that the two ions are prepared in the opposite spin polarizations and their separation is sufficiently short compared to the correlation length of the Coulomb chain [8]. It is the so-called subdecoherent space [8, 9] or decoherence-free subspace (DFS) [10] in which the decoherence can be completely quenched. This is very useful in protecting qubits if they are encoded as 'logical qubits' in two physical qubits [11].

Some studies of the generation of mixed-state entanglement between the two spins in a common harmonic bath have been performed recently [12]. In this paper, we show that the phonon bath can mediate maximal entanglement between the two ions within the DFS. This means that a pure maximally entangled state, with long coherence times, can be generated even in a 'noisy' environment. This robust entangled state could be very useful in the ion-trap quantum computing, particularly on the issue of its scalability. Indeed, a considerable amount of effort has been devoted to physical realization of a scalable ion-trap quantum computer [4] since the first proposal of the quantum computer with trapped ions [13].

We propose to use the ion chain to perform short-ranged quantum communication which is an important element of building up a large-scale quantum computer. According to the blueprint of building a large-scale quantum computer, a quantum computer composed of a number of quantum registers is envisaged [11]. Each quantum register is connected through a common quantum data bus. Quantum gates can be performed in the individual quantum registers and the different registers can communicate with each other through some quantum channels. Typically, this quantum channel will be optical, but alternatives, such as physically transporting the stationary qubits [14] or otherwise using information propagation in a chain of qubits [15] are worth studying as they enable one to avoid the issue of interfacing different types of physical systems.

We consider a situation in which ions in distinct quantum registers (in the veritcal direction) are arranged in different zones [11] and they are interconnected via a chain of ions (in the horizontal direction) as shown in figure 2. Ion-trap technology is developing towards storing ions in multi-trapping zones [16]. *Multi-wafer traps* have been proposed recently to implement the 'X' [17] and 'T' [18] junctions. Thus, the architecture shown in figure 2 is possible. Assume different places for registers and data bus so that the 'T' junction is feasible. The Coulomb ion chain (the horizontal line in figure 2) is being used as a quantum channel between quantum registers. For example, the quantum state of any ion in the quantum register can be transmitted to an ion in the register through transverse phonon modes [19] or the ions can be shuttled from the data bus to the registers in a 'T' junction ion trap [18]. The quantum information can then be teleported [20] through the ion chain using the entanglement between two ions of

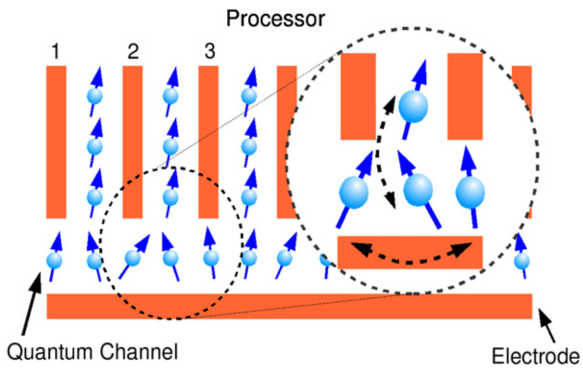

Figure 2. The quantum registers are placed in different zones (in the vertical direction). The different quantum registers are connected by a chain of ions which is served as a quantum channel (in the horizontal direction). An enlarged diagram shows that the quantum state of ions in the quantum register can be transferred to an ion in the chain (dashed arrow indication). The quantum information is then transmitted through the channel.

the chain developed in accordance with the scheme of this paper. In this way, the quantum communication between the different quantum registers can be accomplished. This method offers an alternative to physical shuttling of ions over long distances (the communication mechanism here being through the bath provided by the chain of ions). Consequently, the need for 'segmented' electrodes (necessary for shuttling) is also not a requirement in this method.

We consider N ions arranged in a 1D Coulomb chain. The N ions are confined in a linear trap and interact with each other via the Coulomb repulsion. The trapping and the Coulomb potentials are of the form [3, 21, 22]

$$V_{\text{trap}} = \frac{1}{2}m\sum_{i=1}^{N}\left(\omega_x^2x_i^2 + \omega_y^2y_i^2 + \omega_z^2z_i^2\right),$$

$$V_{\text{Coul}} = \sum_{i>j}^{N}\frac{e^2}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}},$$

where ω_α are the trapping frequencies in the direction $\alpha = x, y, z$; m and e are the mass and the charge of each ion, respectively. The ions form a linear chain along the z-direction for $\omega_{x,y} \gg \omega_z$. The motion of N ions in the 1D chain is described by collective modes, and the dispersion relation is subject to the trapping condition. The approximate eigenvalues of the axial modes can be found as $\omega_n = \omega_z\sqrt{n(n+1)/2}$ for $N \gg 1$ [22]. This approximation is indeed very good for the low axial modes even in a chain of ten ions [22].

The two ions in the 1D Coulomb chain are illuminated by two laser fields individually; the ions interact with the standing wave [3, 4]. The internal states of two ions in the chain are then coupled to the axial vibrations of the entire chain. The Hamiltonian of the ions and the phonon bath reads as ($\hbar = 1$)

$$H_{\text{ion}} = \frac{\omega}{2}\sum_{j=1}^{2}\sigma_z^j, \tag{1}$$

$$H_B = \sum_{n=1}^{N}\omega_n b_n^\dagger b_n, \tag{2}$$

where ω and σ_z^j are the energy splitting and Pauli operator, ω_n and b_n are the frequency and the annihilation operator of the nth phonon mode. The Hamiltonian, in a rotating frame with the laser frequency ω_L, is written as [23]

$$H_L = \sum_{j=1}^{2} \frac{\Omega}{2i}\left(\sigma_+^j + \sigma_-^j\right)\left(e^{i\tilde{k}Z_j} - e^{-i\tilde{k}Z_j}\right), \tag{3}$$

where Ω is the standing wave laser Rabi frequency, $\tilde{k}$ is the wave number of the laser, $Z_j = \sum_n \tilde{z}_n (b_n^\dagger e^{ikz'_j} + b_n e^{-ikz'_j})$ is the position operator of the chain. The parameters k and $\tilde{z}_n$ are $2\pi l/L$ and $1/\sqrt{2m\omega_n}$, respectively, for $l = 0, 1, 2, \ldots, N-1$, $L = Na$ is the length of the chain, and a is the separation between two neighboring ions [22].

In the Lamb–Dicke limit, the total Hamiltonian can be approximated as [23]

$$H \approx \frac{\Delta}{2}\sum_{j=1}^{2}\sigma_z^j + \sum_{n=1}^{N}\left[\omega_n b_n^\dagger b_n + \sum_{j=1}^{2}\sigma_x^j\left(g_n^j b_n^\dagger + g_n^{j*} b_n\right)\right], \tag{4}$$

where $\Delta = \omega - \omega_L$, $g_n^j = \Omega\tilde{k}\tilde{z}_n e^{ikz'_j}$ and $r = z'_1 - z'_2$. It is customary to change the basis as $\sigma_x(\sigma_z) \to \tilde{\sigma}_z(\tilde{\sigma}_x)$ to transform as the conventional form of the spin-boson model [1].

The two spin-boson model can be solved exactly if the parameter Δ is zero [2, 8]. In order to get more insight, we first study the Hamiltonian H_0 in which we set $\Delta = 0$ in equation (4). We apply a canonical transformation $e^S = \exp\left[\sum_{j=1}^{2}\sum_n \tilde{\sigma}_z^j \left(g_n^j b_n^\dagger - g_n^{j*} b_n\right)/\omega_n\right]$ to the Hamiltonian H_0 [1]. In the rotating frame, the Hamiltonian is written as (we have omitted the constant term)

$$\tilde{H}_0 = \sum_{n=1}^{N}\omega_n b_n^\dagger b_n - \lambda\tilde{\sigma}_z^1\tilde{\sigma}_z^2, \tag{5}$$

where $\lambda = 2\sum_{n=1}^{N}\left|g_n^1\right|^2\cos(kr)/\omega_n$. Here we consider that the separation between the two qubits is sufficiently short compared to the correlation length of the bath so that the two qubits are effectively coupled to the common bath [8]. Roughly speaking, the correlation length of the Coulomb chain is about the chain length L if only the low-lying excited modes are involved. Hence, we can impose a condition that the separation r must be less than the length of the chain L. If this condition is satisfied and our procedure ensures that only the low-lying modes are involved, then the two ions will 'feel' to interact with the same harmonic bath.

The bath can be characterized by the spectral density function $J(\omega) = \sum_{n=1}^{N} 2\left|g_n^1\right|^2\delta(\omega - \omega_n)$. For a linear Coulomb chain, in the low-lying excitation regime, the spectral density $J(\omega)$ for a single ion has been shown to be Ohmic and subOhmic when interacting with the traveling wave and the standing wave, respectively [3]. If the ions are equally spaced with a distance a, then the spectral density of the standing wave has the form for $\omega \geqslant \omega_z$ [3],

$$J(\omega) = \eta\omega^{-1}, \tag{6}$$

where $\eta = \Omega^2\tilde{k}^2/m\nu$ and $\nu = \sqrt{3e^2/m\omega_z^2 a^3}$. The spectral density $J(\omega)$ is equal to zero for $\omega < \omega_z$. The above spectral density has been justified by Porras *et al* [3]. Here we remark that the spectral density is obtained from the eigenvalues of axial modes for the large N limit. This approximation is valid with the leading order in $\log N$ for the finite-size systems such as about 50 ions [22].

Now we study the dynamics of internal states of trapped ions interacting with a thermal phonon bath described above. We consider that the trapped ions and the bath are separable initially, i.e., $\rho_T(0) = \rho(0)\otimes\rho_B$, $\rho(0)$ and ρ_B are the density matrix of the qubits and the thermal bath, respectively. The thermal bath is in equilibrium and its density matrix is given by $\rho_B = \mathrm{e}^{-\beta H_B}/Z_B$, where $\beta = 1/k_B T$; k_B and T are the Boltzmann constant and the temperature, respectively, and $Z_B = \mathrm{Tr}\,\mathrm{e}^{-\beta H_B}$ is the partition function.

The reduced density matrix of the two qubits can be obtained by tracing out the system of bath, i.e., $\rho(t) = \mathrm{Tr}_B[\rho_{\mathrm{T}}(t)]$. The reduced matrix of the two qubits is spanned by the basis $\{|1\rangle = |11\rangle, |2\rangle = |10\rangle, |3\rangle = |01\rangle, |4\rangle = |00\rangle\}$. Since the population number is conserved for $\Delta = 0$, the evolution of the diagonal elements of the density matrix is constant: $\rho_{ii}(t) = \rho_{ii}(0)$. The matrix elements satisfy $\rho_{ij} = \rho_{ji}^*$. The non-diagonal matrix element decays as [8]

$$\begin{aligned} \rho_{12}(t) &= \rho_{12}(0)\,\mathrm{e}^{-\mathrm{i}\phi_- - \Gamma}, & \rho_{13}(t) &= \rho_{13}(0)\,\mathrm{e}^{-\mathrm{i}\phi_- - \Gamma}, \\ \rho_{24}(t) &= \rho_{24}(0)\,\mathrm{e}^{-\mathrm{i}\phi_+ - \Gamma}, & \rho_{34}(t) &= \rho_{34}(0)\,\mathrm{e}^{-\mathrm{i}\phi_+ - \Gamma}, \end{aligned} \tag{7}$$

where

$$\phi_\pm = \pm 2\lambda t \pm \int_0^\infty \frac{J(\omega)}{\omega^2} \coth\left(\frac{\omega}{2k_B T}\right) \sin\omega t \,\mathrm{d}\omega, \tag{8}$$

$$\Gamma = \int_0^\infty \frac{J(\omega)}{\omega^2} \coth\left(\frac{\omega}{2k_B T}\right)(1 - \cos\omega t)\,\mathrm{d}\omega. \tag{9}$$

The states under the effect of the collective decoherence read as [8]

$$\rho_{23}(t) = \rho_{23}(0)\,\mathrm{e}^{-2\Gamma_-}, \qquad \rho_{14}(t) = \rho_{14}(0)\,\mathrm{e}^{-2\Gamma_+}, \tag{10}$$

where

$$\Gamma_\pm = \int_0^\infty \frac{J(\omega)}{\omega^2} \coth\left(\frac{\omega}{2k_B T}\right)(1 - \cos\omega t)[1 \pm \cos(kr)]\,\mathrm{d}\omega. \tag{11}$$

The decay rates of the qubits are dramatically changed due to the collective decoherence. For the states ρ_{14} and ρ_{41}, the decay process is greatly enhanced with the decay rate Γ_+. In contrast, the decay rate of the states ρ_{23} and ρ_{32} is largely reduced. In the limit of zero separation r, the states $|01\rangle$ and $|10\rangle$ are protected within the DFS [9]. Physically speaking, the notion of DFS holds if the decay rate of this subspace is comparable or even longer than the coherence times of the ion. In fact, we can estimate explicitly the coherence lifetime of this subspace $\sim\Gamma_-^{-1}$ which is roughly equal to $\left(\eta\omega_c^3 r^2/\omega_z^5 L^2\right)^{-1}$ [1] for low-temperature regime and ω_c is the cut-off frequency. Therefore, the long coherence lifetime can be achieved for the small separation $r \ll L$. For convenience, we have introduced the cut-off frequency ω_c to the spectral density in equation (6) of the form $J(\omega) = \eta\omega^{-1}\,\mathrm{e}^{-\omega/\omega_c}$ since it is legitimate to perform the adiabatic elimination of the high-frequency modes compared to the tunneling strength Δ [1]. Nevertheless, it is required that the cut-off frequency ω_c is much greater than the laser strength Δ and the thermal energy $k_B T/\hbar$ [1]. This treatment helps us to find out the bound of the integral (see the footnote) and gives a rough estimation for the decaying rate.

Now we investigate the effect of the two local laser fields on the two qubits for a nonzero tunneling parameter Δ. The transformed Hamiltonian $\tilde{V} = \mathrm{e}^S V \mathrm{e}^{-S}$ can be obtained:

$$\tilde{V} = \frac{\Delta}{2}\sum_{j=1}^{2}\left(\tilde{\sigma}_+^j B_j^\dagger + B_j \tilde{\sigma}_-^j\right), \tag{12}$$

where $B_j = \exp\left[\sum_n \left(g_n^j b_n^\dagger - g_n^{j*} b_n\right)/\omega_n\right]$ and $B_1^\dagger B_2 \approx \mathbf{I}$ for a small separation r.

[1] The decay rate Γ_- is bounded by $(k_c r)^2 I$ for $k_c r \ll 1$, $I = \int_0^\infty J(\omega)/\omega^2 \coth(\omega/2k_B T)(1 - \cos\omega t)\,\mathrm{d}\omega$ and the wave number $k_c \approx 2\pi\omega_c/\omega_z L$ is chosen at the cut-off frequency. The bound of integral $I \leqslant 2\eta\coth(\omega_z/2k_B T)(\omega_c - \omega_z)/\omega_z^3$ can be found for the subOhmic spectral density in equation (6).

We consider the tunneling strength Δ is much weaker than the spin-bath coupling λ. This enables us to derive an effective Hamiltonian within the DFS based on the second-order perturbation theory. The matrix elements of the effective Hamiltonian can be represented as

$$(\tilde{H}_{\text{eff}})_{mn} = -\sum_{l} \frac{\tilde{V}_{ml}\tilde{V}_{ln}}{E_l^{(0)} - \left(E_m^{(0)} + E_n^{(0)}\right)/2}, \tag{13}$$

where m and n represent the basis of unperturbed states of $\tilde{\sigma}_z^j$: $\{|01\rangle, |10\rangle\}$, and l denotes the intermediate states $\{|11\rangle, |00\rangle\}$; $E_l^{(0)}$ are the eigenenergies of $\tilde{H}_0$.

The effective Hamiltonian can be obtained as [24]

$$\tilde{H}_{\text{eff}} = \kappa(J_+J_- + J_-J_+), \tag{14}$$

where $\kappa = \Delta^2/8\lambda$ and $J_\pm = \sum_{j=1}^{2}\tilde{\sigma}_\pm^j$. The dynamics within the DFS is governed by this effective Hamiltonian. The state with the initial state $|10\rangle$ evolves

$$|\Psi(t)\rangle = \mathrm{e}^{-2\mathrm{i}\kappa t}[\cos 2\kappa t|10\rangle - \mathrm{i}\sin 2\kappa t|01\rangle]. \tag{15}$$

The two qubits become entangled when the two local laser fields are turned on. At the time $t^* = \pi/8\kappa$, up to a global phase factor $\mathrm{e}^{\mathrm{i}\pi/4}$, the quantum state becomes $|\Psi(t^*)\rangle = (|10\rangle - \mathrm{i}|01\rangle)/\sqrt{2}$.

An ideal entangled state is then generated dynamically. Here the spin-coupling strength can be estimated to be 10^4–10^5 Hz $\left(\lambda \leqslant \Omega N\omega_z^{-1/2}\right)$; for the typical values of the laser Rabi and trapping frequencies are around several MHz and 10^5 Hz, respectively [25]. The tunneling strength, $\Delta = \omega - \omega_L$, can be adjusted by choosing an appropriate frequency of the laser but it has to be much smaller than the spin-bath coupling λ. Therefore, the speed of entanglement formation can be estimated around 10^3–10^4 Hz. The rate of entanglement generation can be increased by a stronger laser. Although the ions are inevitable to the decoherence such as spontaneous emission [26], we can ignore the other noise sources to the ions. Since the ions are with long coherence times (up to 100 ms) [27], it is very long compared to the timescale of entanglement generation.

We note that the entanglement generation shares the same spirit of the Sørensen–Mølmer entangling gate in which the entanglement gate is implemented by coupling to the virtual motional states of a '*single*' center-of-mass (CM) mode [28]. Indeed, the Sørensen–Mølmer gate has been shown to entangle two $^{40}Ca^+$ ions with a bichromatic laser [25]. However, it is inevitable to couple a number of phonon modes for a chain of ions in performing the gate operation with a traveling wave as considered by Jonathan and Plenio [29]. However, we have studied a more general scenario by the consideration of position-dependent couplings to the phonon bath in which the collective decoherence effect sets in. The entanglement can be efficiently generated within the DFS. The fidelity between the generated and ideal entangled states $|\Psi(t^*)\rangle$ is $\mathrm{e}^{-\Gamma_-(t^*)} \approx 1 - \Gamma_-(t^*)$. The rate, $\Gamma_-(t^*)$, is roughly proportional to the factor $(r/L)^2$ explicitly (see the footnote). It thus limits the separation for entangling two ions. For a chain of ions containing 50–100 ions, the high fidelity of the entangled pair can be generated between the two ions separate from a few ions, say five ions, in the chain.

Having discussed the generation of the entangled state, we proceed to study the quantum communication between two trapped ions. This can be used to transfer quantum information between different quantum registers by the implementation of the quantum teleportation protocol [20]. For instance, we consider to transmit the quantum information from ion i to ion j. We first generate the entanglement between the ions j and k. Ion k is next to ion i. Then, we perform the Bell-state measurement between the ions i and k. The quantum teleportation can thus be accomplished by sending the measurement result to ion j through the classical communication. It is noteworthy that the range of quantum state transfer should be short in our scheme. However, this can be resolved by repeating the quantum teleportation between two nearby ions several times until the quantum information being transferred to that distant ion. Alternatively, we can entangle the two distant ions with a relatively low fidelity of the maximal entanglement. We can apply entanglement purification (if the fidelity is higher than 0.5 [30]) that has been demonstrated in the ion-trap experiment [31]. Our scheme benefits from without the direct transport of ions.

Additionally, the ion chain can form a cluster state [32] by sequential generation of entangled states between different pairs of ions, with the entangling of each pair taking place according to the scheme of this paper, which will be useful for measurement-based quantum computing.

In conclusion, we have investigated the two ions coupled to axial vibration modes of the 1D Coulomb chain. This can be shown to be equivalent to the 'two spin-boson' model. We show that the decoherence-free entanglement of two nearby ions can be dynamically generated. It can be applied to perform short-ranged quantum communication in ion traps doing away with the necessity of physically shuttling the ions from place to place. The present result is not confined to the ion traps system and could also benefit other quantum information processing in solid-state-based systems which can be described by the spin-boson model such as the Josephson charge qubits of a Cooper-pair box [33] and semiconductor double quantum dots [34].

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References

[1] Leggett A J *et al* 1987 *Rev. Mod. Phys.* **59** 1
[2] Breuer H P and Petruccione F 2002 *The Theory of Open Quantum Systems* (Oxford: Oxford University Press)
[3] Porras D *et al* 2008 *Phys. Rev.* A **78** 010101(R)
[4] Leibfried D *et al* 2003 *Rev. Mod. Phys.* **75** 281
[5] Diedrich F *et al* 1987 *Phys. Rev. Lett.* **59** 2931
Blëmel R *et al* 1998 *Nature* **334** 309
[6] Itano W M *et al* 1998 *Science* **279** 686
Mitchell T B *et al* 1998 *Science* **282** 1290
[7] Schätz T, Schramm U and Habs D 2001 *Nature* **412** 717
[8] Palma G M *et al* 1996 *Proc. R. Soc. Lond.* A **452** 567
[9] Duan L M and Guo G C 1997 *Phys. Rev. Lett.* **79** 1953
[10] Lidar D A *et al* 1998 *Phys. Rev. Lett.* **81** 2594
[11] Kielpinski D, Monroe C and Wineland D J 2002 *Nature* **417** 709
[12] Braun D 2002 *Phys. Rev. Lett.* **89** 277901
[13] Cirac J I and P Zoller P 1995 *Phys. Rev. Lett.* **74** 4091
[14] Rowe M A *et al* 2002 *Quantum Inf. Comput.* **2** 257
[15] Bose S 2007 *Contemp. Phys.* **48** 13
[16] Amini J M *et al* 2008 arXiv:0812.3907
[17] Blakestad R B *et al* 2009 arXiv:0901.0533
[18] Hensinger W K *et al* 2006 *Appl. Phys. Lett.* **88** 034101
[19] Zhu S L, Monroe C and Duan L M 2006 *Phys. Rev. Lett.* **97** 050505
[20] Bennett C H *et al* 1993 *Phys. Rev. Lett.* **70** 1895
[21] Porras D and Cirac J I 2006 *Phys. Rev. Lett.* **96** 250501

[22] Morigi G and Fishman S 2004 *Phys. Rev. Lett.* **93** 170602
Morigi G and Fishman S 2004 *Phys. Rev.* E **70** 066141
[23] Blatt R, Cirac J I and Zoller P 1995 *Phys. Rev.* A **52** 518
Helon C D and Milburn G J 1996 *Phys. Rev.* A **54** R25
[24] Ng H T, Law C K and Leung P T 2003 *Phys. Rev.* A **68** 013604
[25] Kirchmair G *et al* 2009 *New J. Phys.* **11** 023002
[26] Mintert F *et al* 2005 *Phys. Rep.* **415** 207
[27] Benhelm J *et al* 2008 *Phys. Rev.* A **77** 062306
[28] Sørensen A and Mølmer K 1999 *Phys. Rev. Lett.* **82** 1971
[29] Jonathan D and Plenio M B 2001 *Phys. Rev. Lett.* **87** 127901
[30] Bennett C H *et al* 1997 *Phys. Rev. Lett.* **78** 2031
[31] Reichle R *et al* 2006 *Nature* **19** 838
[32] Raussendorf R and Briegel H J 2001 *Phys. Rev. Lett.* **86** 5188
[33] Nakamura Y, Pashkin Yu A and Tsai J S 1999 *Nature* **398** 786
Makhlin Y, Schön G and Shnirman A 2001 *Rev. Mod. Phys.* **73** 357
[34] Hayashi T *et al* 2003 *Phys. Rev. Lett.* **91** 226804